Hedging Cryptos with Futures

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joint work with
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26 April 2021
Overview

Motivation

Copula-based hedging

Data

Results
Digital assets are here to stay

- Markets for cryptocurrencies are maturing:
  - Institutional investors are buying into it.
  - Regulators are working hard to make stablecoins “safe” (e.g. resolve issues of jurisdiction, financial stability).
  - Exchanges (e.g. CME) are issuing futures and options.

⇒ We are in the middle of the digitalisation of financial markets...
Digital assets are here to stay

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- Institutional investors are buying into it.
- Regulators are working hard to make stablecoins “safe” (e.g. resolve issues of jurisdiction, financial stability).
- Exchanges (e.g. CME) are issuing futures and options.

⇒ We are in the middle of the digitalisation of financial markets... (... and it’s progressing rapidly.)
Digital assets are here to stay

As Bitcoin Rises, Institutions Make Crypto Market Impact

Barriers fall away but hedging remains a challenge; regulatory clarity will help

Friday, February 26, 2021

By John Hintze

https://www.garp.org/#!/risk-intelligence/market/investment-management/a1Z1W000005kZDGUA2

Motivation

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Bitcoin futures

CME launched Bitcoin Futures in December 2017 and options on futures in January 2020

Bitcoin Future:
- Underlying: Bitcoin Reference Rate (BRR), based on relevant bitcoin transaction on certain exchanges
- Maturities: nearest two Decembers and nearest six consecutive months
- Settlement: cash

https://www.cmegroup.com/trading/equity-index/us-index/bitcoin.html
Hedging cryptos

- Hedging Bitcoin exposure with Bitcoin futures:
  - Basis risk
  - BRR not traded
  - Ability of futures to hedge tail risks?

Source: skew.com, December 2019

Two directions:
- Copulas
- Risk measures
Hedging cryptos

- Hedging Bitcoin exposure with Bitcoin futures:
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- Hedge other cryptos with Bitcoin?
  - High correlation
  - Tail risks, extreme events?

Source: skew.com, December 2019

Motivation  N. Packham  6
Hedging cryptos

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Hedging cryptos

**Hedging Bitcoin exposure with Bitcoin futures:**

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**Hedge other cryptos with Bitcoin?**

- High correlation
- Tail risks, extreme events?

**Two directions:**

- Copulas
- Risk measures

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**1M Correlation Matrix**

```
<table>
<thead>
<tr>
<th>Index</th>
<th>BTC</th>
<th>ETH</th>
<th>XRP</th>
<th>USDT</th>
<th>BCH</th>
<th>LTC</th>
<th>EOS</th>
<th>BNB</th>
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<td>BTC</td>
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<td>91.17%</td>
<td>81.77%</td>
<td>-15.59%</td>
<td>88.69%</td>
<td>88.85%</td>
<td>90.70%</td>
<td>84.47%</td>
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<tr>
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<td>91.17%</td>
<td>100.00%</td>
<td>78.50%</td>
<td>-24.51%</td>
<td>90.18%</td>
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<tr>
<td>XRP</td>
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<td>78.50%</td>
<td>100.00%</td>
<td>-8.21%</td>
<td>81.90%</td>
<td>81.68%</td>
<td>84.96%</td>
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<tr>
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<td>-16.64%</td>
<td>-18.98%</td>
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<tr>
<td>BCH</td>
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<td>87.02%</td>
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<td>95.78%</td>
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<td>78.61%</td>
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<td>BSV</td>
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<td>26.99%</td>
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<td>24.13%</td>
<td>34.09%</td>
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</tbody>
</table>
```

Source: skew.com, December 2019
Overview

Motivation

Copula-based hedging

Data

Results
Hedging spot with futures

- Hedge portfolio return: $R_t^h = R_t^S - h R_t^F$, where
  - $R_t^S$: spot return at time $t$
  - $R_t^F$: futures return at time $t$
  - $h$: hedge ratio

- Goal: Find optimal hedge ratio $h^*$

- Minimum-variance hedge ratio, e.g. Ederington (1979), assumes variance as risk measure and elliptical return distribution

- Extensions: risk measures, copulas, e.g. (Harris and Shen, 2006; Barbi and Romagnoli, 2014)
Definition

A (bivariate) **copula** is a distribution function on $[0, 1]^2$ with standard uniform marginals.

- Copulas differ only through the dependence between the marginals.
- Sklar’s Theorem (below) captures that copulas allow to separate
  - modelling of the marginals, and
  - modelling of the dependence structure.
Theorem (Sklar’s Theorem)

Let $F$ be a joint distribution function with margins $F_1, F_2$. Then, there exists a copula $C : [0, 1]^2 \rightarrow [0, 1]$ such that, for all $x, y \in \mathbb{R}$

$$F(x, y) = C(F_1(x), F_2(y)).$$  \hspace{1cm} (1)

If the margins are continuous, then $C$ is unique; otherwise $C$ is unique on the range of the margins.

Conversely, if $C$ is a copula and $F_1, F_2$ are univariate distribution functions, then the function $F$ defined by (1) is a joint distribution function with margins $F_1, F_2$.

Representation of $C$ in terms of $F$ and its margins:

$$C(u, v) = F(F_1^{-1}(u), F_2^{-1}(v)).$$
Examples of copulas

- Gaussian
- Student t
- Clayton
- Frank
- Gumbel
- Plackett
- NIG Factor
- Mixture of Gaussian and Product

▶ All copulas are calibrated to a Spearman’s Rho of 0.75.
Copula-based hedging

Proposition (Barbi and Romagnoli (2014))

Let $X$ and $Y$ be two real-valued random variables with corresponding absolutely continuous copula $C$ and continuous marginals $F_X$ and $F_Y$. Then, the distribution of of $Z = X - hY$ is given by

$$F_Z(x) = 1 - \int_0^1 D_1 C \left[ u, F_Y \left\{ \frac{F_X^{-1}(u) - x}{h} \right\} \right] \, du.$$

Easy to show (e.g. McNeil et al. (2005)):

$$D_1 C(F_X(x), F_Y(y)) = \frac{\partial}{\partial u} C(u, v) = P(Y \leq y | X = x).$$
Risk measures

- **Variance**: \( \text{Var}(Z) \)

- **Value-at-risk (VaR)**: \( \text{VaR}_\alpha = -F_Z^{(-1)}(1 - \alpha) \)

- **Expected Shortfall (ES)**: \( \text{ES}_\alpha = -\frac{1}{1 - \alpha} \int_0^{1-\alpha} F_Z^{(-1)}(p)dp \).
Risk measures

▶ **Spectral risk measures (SRM)** (Acerbi, 2002; Cotter and Dowd, 2006):

\[
\rho_\phi = - \int_0^1 \phi(p) F_Z^{(-1)}(p) \, dp,
\]

where \( q_p \) is the \( p \)-quantile of the return distribution and \( \phi(s) \), \( s \in [0, 1] \), is the so-called **risk aversion function**, a weighting function such that

(i) \( \phi(p) \geq 0 \),

(ii) \( \int_0^1 \phi(p) \, dp = 1 \),

(iii) \( \phi'(p) \leq 0 \).

▶ SRM’s are coherent risk measures.
Risk measures

▶ **Exponential spectral risk measure**: weighting function \( \phi(p) = \lambda e^{-k(1-p)} \), where \( \lambda \) is an unknown positive constant, derived from exponential utility function:

\[
\rho_\phi = \int_0^1 \phi(p) F_{Z^{-1}}(p) \, dp = \frac{k}{1 - e^{-k}} \int_0^1 e^{-k(1-p)} F_{Z^{-1}}(p) \, dp.
\]
Optimal hedge ratio

- Hedge portfolio: \( R_t^h = R_t^S - hR_t^F \), with \( h \) hedge ratio

- Optimal hedge ratio:
  \[
  h^* = \arg\min_h \rho(h),
  \]
  where \( \rho(h) \) is the risk of the hedge portfolio with hedge ratio \( h \).
Overview

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Copula-based hedging

Data

Results
Data

- Daily log returns, 23pm CET
- 29 May 2018 through 3 Feb 2021
- Spot: Coingecko Bitcoin / USD
- Future: CME BTC Future
- Sources: Bloomberg, coingecko
CRIX returns with 30 most extreme observations

Return CRIX minus BTC future

▶ I.e., hedge ratio of 1.
Student $t$ distribution: $\nu = 7.95$

Generalised Pareto distribution (EVT): tail index $1/\xi = 4.92$
Spot and Future

BTC Future

CRIX

Data N. Packham
Spot and Future empirical copula

Data

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Overview

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Copula-based hedging

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Results
Calibration

- Calibration of eight copula models (see slide Copulas) via method of moments, see e.g. (Genest and Rivest, 1993; Oh and Patton, 2013)

- “Moments”:
  - Spearman’s Rho
  - Quantile dependence at 0.05, 0.1, 0.9, 0.95 quantiles

- Margins follow empirical distribution

- Recalibration of copula and optimisation of $h^*$ every 5 days with 300 data points
Optimal hedge parameters

![Graphs of optimal hedge parameters for different distributions: Gaussian, t Copula, Clayton, Frank, Gumbel, Plackett, NIG Factor, Gauss Mix Indep. Each graph shows the variance, VaR 99%, VaR 95%, ES 99%, ES 95%, and Spectral 10 metrics over time.]

Results

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Hedge Effectiveness

- Hedge effectiveness (Ederington, 1979) captures percentage reduction in risk:

\[ 1 - \frac{\rho(r^h)}{\rho(r^S)}. \]

- Optimisation of \( h^* \) every 5 days based on 300-day-window.
- Out-of-sample effectiveness calculated out-of-sample on 100 day window.
Hedge effectiveness

Out-of-Sample Hedging Effectiveness of Variance

Results
Hedge effectiveness

Out-of-Sample Hedging Effectiveness of VaR 99%

Results

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Hedge effectiveness

Out-of-Sample Hedging Effectiveness of VaR 95%

Results

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Hedge effectiveness

Out-of-Sample Hedging Effectiveness of ES 99%

Results

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Hedge effectiveness

Out-of-Sample Hedging Effectiveness of ES 95%

Results
Hedge effectiveness

Out-of-Sample Hedging Effectiveness of ERM k=10

Results

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P&L (dynamic)

CRIX returns with 20 most extreme observations

BTC future returns with 20 most spot extreme observations
P&L (dynamic)

- Daily return from hedge, out-of-sample
- Recalibration every 5 days

Results

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P&L (dynamic)

- Daily return from hedge, out-of-sample
- Recalibration every 5 days

Results

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- RMSE (compared to zero) relative to RMSE of unhedged position
- 99% risk measures underperform
- Downside semi-variance relative to downside semi-variance of unhedged position
- VaR 95% and SRM perform best
P&L (static)

P&L from static hedge, out-of-sample, 100 days, rolling every 5 days
P&L (static)

- P&L from static hedge, out-of-sample, rolling every 5 days
- From left to right: Variance, VaR 99%, VaR 95%, ES 99%, ES 95%, Spectral 10

Results

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- RMSE (compared to one) relative to RMSE of unhedged position
- Gumbel with ES 95% or SRM performs best
- Downside semi-variance relative to downside semi-variance of unhedged position
- SRM performs best, together with VaR 95%

Results

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Conclusion

- Hedging with different copulas and risk measures produces mixed results:
  - Frank copula underperforms consistently
  - NIG and Gaussian Mix produce small hedge ratios pre-Covid-19 pandemic
  - NIG factor produces good hedge effectiveness
  - Gumbel produces good results in P&L

- Next step: Hedge other cryptos (CRIX index) with BTC futures
References


Thank you!

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P&L (dynamic)

- Daily return from hedge, out-of-sample
- Recalibration every 5 days
Daily return from hedge, out-of-sample
Recalibration every 5 days
P&L (dynamic)

- Variance
- VaR 99%
- VaR 95%
- ES 99%
- ES 95%
- Spectral 10

▶ Daily return from hedge, out-of-sample
▶ Recalibration every 5 days
P&L (dynamic)

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